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Carmen Pagliari [1], Nicola Mattoscio[2], Iacopo Odoardi [3], Donatella Furia[4]

INSTABILITY AND INTRINSIC DISORDER IN UNEMPLOYMENT DYNAMICS: A NONLINEAR REVISION OF OKUN'S LAW

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Abstract

We test the significance of nonlinearity in unemployment dynamics by determining the analytical conditions for the existence of stable steady states, bifurcations and deterministic chaos, to investigate the intrinsic origin of irregularity in unemployment fluctuations. Our model, based on Okun's Law, permits deducing the role of labor market flexibility and labor productivity in the control of unemployment and provides policy suggestions. We empirically prove that uncontrollability of unemployment is sufficiently distant in most advanced economies. Nevertheless, using numerical experiments, we determine that there are countries in which unemployment evolves towards numerical disorder.

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1. Introduction

In economic studies, the relation between economic growth and

[1]University of Chieti-Pescara, Pescara (Italy), *E-Mail address*: carmen.pagliari@unich.it.
[2] University of Chieti-Pescara, Pescara (Italy), *E-Mail address*: nicola.mattoscio@unich.it.
[3] Corresponding author**,** University of Chieti-Pescara, Pescara (Italy), *E-Mail address*: iacopo.odoardi@unich.it.
[4] University of Chieti-Pescara, Pescara (Italy), *E-Mail address*: donatella.furia@unich.it.

unemployment is fundamental for suggesting policy measures to achieve macroeconomic objectives (Mankiw 2012; Romer 2012). This relation has been empirically tested and studied for many years, and a deeper knowledge of the involved dynamics should be useful for providing theoretically and empirically based forecasts. A well-known relationship that involves the GDP and the unemployment level was determined by Arthur Melvin Okun (1962), who observed an empirical regulatory in the relationship between these variables in the US. The Law continues to be a reference point for scholars, since This regulatory was proven to be a robust result in time (Blinder 1997) as, e.g., in the contribution of Valadkhani and Smyth (2015), who verified that the relationship was persistent in the United States since the Second World War, although weaker during the recession of the 1980s. For this reason, several studies (Economou and Psarianos 2016; Zanin 2016; Ibragimov and Ibragimov 2017; Guisinger et al. 2018) focused on interesting revisions of Okun's Law also in the post-crisis period.

Our aim is to study an analytical revision of Okun's Law on the basis of nonlinear dynamics, highlighting microeconomic interconnections and fundamental macroeconomic effects. and, as far as we know, few researches exist on the application of this method (see Section 3). We propose a theoretical model that is useful for investigating the endogenous conditions that generate stable steady states, bifurcations and deterministic chaos in unemployment dynamics and apply it to the G7 countries, deducing forecasts for the risk of chaos in two of these countries, the dynamics of which reveal opposite extreme characteristics. We examine possible economic interpretations of the coefficients of our model and the significance of the resulting forecasts. In particular, the coefficients we consider are based on some aspects of the theoretical interdependence between unemployment and GDP that are intrinsic to the system, and therefore they are suitable for an economic interpretation that is different from the explanation usually provided for an econometric coefficient. The nonlinear dynamic approach used for the study of deterministic chaos can appropriately contribute to this research field.

Our study is based on Okun's Law that is a quantitative expression of a persistent phenomenon (see Section 2). We analyze whether the dynamics of the unemployment rate may be characterized by chaos, i.e., being equivalent to disorder and uncontrollability. In our analysis we consider the widely used relation between the growth rate gap and the unemployment rate by formulating several hypotheses about the dynamics of the considered variables and using the logistic map. For the first application of the results of our theoretical model, we focus on the G7 countries (as in Pierdzioch et al. 2011) in the period 1981-2017; in the second application of our model, we consider the data in the preceding eighteen years to analyze forecasts of the ranges of variability of the unemployment rate and compare the values to which the unemployment level should tend. The considered time periods include the recent Great Recession; hence, we show that the model can be used even in the case of a severe negative conjuncture. We expect at least the major economies to show a stronger presence of the Okun's relationship and greater socioeconomic stability. This should involve both avoiding chaos and showing similar levels of unemployment rates to which the economies could trend for the two periods analyzed.

For our research objective, we suggest that the level of complexity of a dynamic system can be analyzed by applying the methods of the mathematics of chaos. The applications of the logistic map that is characterized by nonlinearity and a unimodal representation are particularly useful in this framework. This map is suitable for a simple mathematical expression of the dynamics of a complex system, especially in the circumstances deriving from the presence of dual effects of a single cause that are connected by a chain reaction and responsible for nonlinearity (Rosser Jr. 2000; Strogatz 2014).

The introduction of the paper clarifies our specific research objectives. In Section 2, we discuss some aspects of the Okun's Law with a brief literature review that serves as the basis of our work, in reference to our research objectives. In Section 3, the model is presented, and the hypotheses of our analysis are described to deduce the fundamental nonlinear dynamic equation of the model that can be interpreted as an extension of Okun's Law. In Section 4, the discussion of the numerical consequences of steady states, bifurcations, cycles and chaos in relation to the alternative values of the parameters that characterize the nonlinear dynamics of unemployment is visualized by simple graphical representations. Evidence from the application of our model is highlighted to investigate if chaos is indeed significant in unemployment dynamics; in particular, we prove that the circumstances of deterministic chaos in unemployment dynamics are sufficiently distant in most of the advanced economies. In Section 5, policy suggestions are proposed.

2. Some aspects of Okun's Law

One of the reasons to keep attention to the contribution of A. M. Okun's Law in the 21st century is the analysis and comparison of different values of its coefficient for the forecast and the systemic information that it can provide. We know that this coefficient varies according to the considered variables (e.g., note the difference between the use of the gross domestic income and the GDP in Nalewaik 2010), the analytic technique (Zanin and Marra 2012; Ibragimov and Ibragimov 2017) or the focus of the investigations (see the sectoral coefficients in Basu and Foley 2013; and see Okun's coefficients by age and gender in Zanin 2016). In the economic literature, alternative points of view exist on the consistency and adequacy of forecasts and policy implications derived from models based on Okun's Law. This empirical law is not always verifiable (it has scarce significance for Durech et al. 2014), and its characteristic coefficient changes for each examined context (e.g., the law is observed to hold in the G7 countries, but its coefficient varies according to Moosa 1997). In some cases, the significance of the relationship has been criticized (Meyer and Tasci 2012) in terms of the capacity of explaining economic recoveries (as in Owyang and Sekhposyan 2012) for what happened in the "jobless recoveries" after the last three recessions (see the role of labor force participation and productivity in Gordon 2010), while the law has been confirmed in the same context by other scholars (see Ball et al. 2013). Furthermore, there are studies that prove the validity of Okun's assumption in both the short run and the long run (Huang and Yeh 2013) as well as the high quality of the related forecasts (Ball et al. 2015). However, forecasts are subject to criticism because they are at risk of excessive change in value whenever there is the inevitable revision of the original macroeconomic data (Guisinger and Sinclair 2015). Finally, other criticism concerns the fact that several factors affect the determination of the output gap (see Prachowny 1993).

An important aspect to be considered in studies and tests of the aforementioned relationship is its validity during crisis and recessionary periods. In fact, it is reasonable to wonder if Okun's Law was still valid during the economic recession that followed the international financial crisis in many Western countries. Some scholars have considered a breakdown of the relation after the crisis (IMF 2010; Owyang and Sekhposyan 2012) or past crises (Daly and Hobijn 2010). According to Daly et al. (2014), the supposed breakdown of the relationship in the years of the crisis in the US is caused not only by the quality of real-time data compared to the revised values but also by the lag in the adjustment of employment levels. According to Elsby et al. (2010), the relationship is confirmed in the US at least until early 2009, when the increase in unemployment was combined with an economic recovery influenced, according to the authors, by a contemporaneous strong increase in average productivity, as had occurred during the jobless recoveries that followed the previous two recessions in this country.

In fact, after an economic crisis, job growth could be lower than that expected from the Law (Gordon 2010). In Cazes et al. (2013), tests confirmed Okun's Law during the years of the 2007 financial crisis and that the variations of the relationship were due to the employment protection legislation of each country. For example, the presence of trade unions and collective bargaining makes Okun's relation weaker in many countries (see Izyumov and Vahaly 2002 for analysis of the relevant changes observed in formerly planned economies). Effects on unemployment are different and related to labor market protection expenditure in a relativity homogenous group of countries, as in Europe (Economou and Psarianos 2016). Labor market legislation causes large differences in the macroeconomic dynamics, as demonstrated, among others, by Sögner and Stiassny (2002) and Sarkar (2013).

Other studies (Palley 1993; Virén 2001; Silvapulle et al. 2004) have highlighted an asymmetry in Okun's Law: positive turns in the economic cycle have smaller effects on unemployment than do negative turns. As a result of the above findings, we expect to observe differences among the analyzed countries of the G7 group as a consequence of institutional and structural features. We know that recent applications of Okun's Law have confirmed the original empirical regularities for the US and Canada, highlighting the statistical significance of the same law being weaker in Europe and Japan (see, for an analysis of the G7 countries, Moosa 1997).

Policy suggestions can be proposed consistently with Okun's Law (Blinder 1997), which can be used to make valid forecasts (Mitchell and Pearce 2010). In this regard, there are studies suggesting that economists' forecasts for several advanced economies reflect Okun's Law (Pierdzioch et al. 2011 for the G7 countries and Pierdzioch et al. 2012 for the Eurozone). The contribution of Ball et al. (2015) also proves that forecasts based on the relation between the GDP and the unemployment rate are consistent with Okun's relationship, at least for advanced economies.

3. Unemployment's nonlinear dynamics

In view of our analysis, deterministic chaos is relevant to the study of unemployment dynamics. We observe that fluctuations in unemployment as well as in inflation and in many other macroeconomic variables are often characterized by irregularity. The numerical disorder in the time series of these variables may be due to exogenous or endogenous causes, and the simple observation of time series may be insufficient for the investigation of the nature of these causes. The exogenous origin of irregular fluctuations is usually ascribed to stochastic events that affect the time evolution of the considered variables, while the endogenous origin can be explored in the possible presence of deterministic chaos.

The deterministic approach to economic dynamics is useful for testing the presence of potential endogenous chaos and for testing the characteristics of the future time paths in relation to the initial conditions, that determine the evolution of the economic system. If a model of deterministic dynamics is available, then it can simplify the analysis of the future time-paths of the economic variables over a large number of periods, emphasizing the dimensions of irregularity and unpredictability that are incorporated in the initial and/or actual conditions on which the future of the economic system is endogenously founded.

The significance of deterministic chaos in relation to the endogenous origin of business cycle irregularity has been proven by many studies since the 1980s (see Day 1982 and 1983; Goodwin 1990; Grandmont 1985). Therefore, the effectiveness of the deterministic approach should be examined also in relation to unemployment dynamics because of the strict interdependence between unemployment and business cycles.

A deterministic approach has been used for revising the Phillips curve (see Soliman 1996). Recently, the presence of deterministic chaos has been studied in relation to the dynamics of the trade-off between inflation rate and unemployment rate (Zhang 2006; Flaschel and Proaño 2014); however, very few studies investigated deterministic chaos in the dynamics of the unemployment rate based on Okun's approach (Jablanović 2011 and 2014).

The dynamic model that founds the analysis contained in this paper is based on five hypotheses that involve the growth rate gap (between the real output and its potential level) and the unemployment rate. The real output is the real GDP, and its potential value derives from IMF evaluations[5]. These dynamical assumptions, referred to as H1 ÷ H5, lead to a generalized nonlinear version of Okun's Law that will be explained in the following part of this Section.

A traditional form of Okun's Law can be expressed as follows (Dornbush

[5] These values are calculated using the "Output gap in percentage of potential GDP" measure and the real GDP of the same year (source: IMF data).

and Fisher 1985, p. 483): $u_{t+1} = u_t - \gamma(g_{t+1} - g^e_{t+1})$. In this form, Okun's Law states that the unemployment rate u declines by γ percentage points for every 1 percentage point of annual real output growth g above its potential level. On the other hand, the same law establishes that if output growth were 1% below its potential level, unemployment would rise by γ percentage points. In the formula, u_t is the unemployment rate at time t, while g_{t+1} and g^e_{t+1} are, respectively, the growth rate of real output at time $(t+1)$ and the rowth rate of the potential output at the same time, and γ is a real positive number (a parameter).

As the basis of our model, we assume the following five hypotheses (see Pagliari and Mattoscio 2019 for a general formalization of this model and its graphical representation):

H1 - There exists a linear relation between the growth rate gap at time $(t+1)$ and the contemporaneous (i.e. in the same time) unemployment rate. The relation can be written as follows:

$$|g_{t+1} - g^e_{t+1}| = \rho u_{t+1} \qquad (\rho > 0) \tag{1}$$

H2 - The growth rate gap is a function of the unemployment rate of the previous time:

$$|g_{t+1} - g^e_{t+1}| = f(u_t) \tag{2}$$

H3 - Function $f(u_t)$ is composed of two additive components that express two effects connected by a chain reaction and defined in the following hypotheses H4 and H5:

$$f(u_t) = D(u_t) + I(u_t) \tag{3}$$

H4 - Component $D(u_t)$ expresses a direct linear effect on the growth rate gap of time $(t+1)$ due to the unemployment rate of time t:

$$D(u_t) = \alpha_D u_t \qquad (\alpha_D \neq 0) \tag{4}$$

H5 - Component $I(u_t)$ expresses an indirect nonlinear (quadratic) effect on the growth rate gap of time $(t+1)$ indirectly arising from the effect of the unemployment rate of time t on the contemporaneous aggregate demand:

$$I(u_t) = \alpha_I u_t^2 \qquad (\alpha_I \neq 0) \tag{5}$$

Hypotheses H2, H3, H4, and H5, considered simultaneously, lead to the following dynamic relation:

$$|g_{t+1} - g^e_{t+1}| = \alpha_D u_t + \alpha_I u_t^2 \tag{6}$$

It is most significant to hypothesize a quadratic dependence of the absolute value of the real output growth gap $(g_{t+1} - g^e_{t+1})$ on the unemployment rate u_t of the previous time. This statement is supported by some contributions in the economic literature. In fact, the nonlinearity of Okun's relationship has been extensively tested in theoretical and applied studies (see Cuaresma 2003); additionally, other studies prove nonlinearity in unemployment and in labor markets both by theoretical (see Dufourt et al. 2008) and by empirical (see Panagiotidis and Pelloni 2007) approaches. Moreover, the observation of real data often shows a nonlinear relation between the GDP growth rate gap and the unemployment rate of the previous year, according to hypothesis H5[6].

In relation to hypotheses H4 and H5, we refer to a double effect of the unemployment rate on the growth rate gap of the real output. There is a direct effect induced on the level of the output growth rate gap by the direct technological link between the quantity of production and the labor factor. Moreover, there is an indirect effect on the level of the gap through the aggregate demand; this latter effect is influenced by the individual disposable income that originates from the level of production dependent on the level of the unemployment rate. These microeconomic and macroeconomic effects are the basis of nonlinearity in our theoretical model that is proposed for testing the presence of endogenous and intrinsic irregularity in unemployment dynamics.

Considering equalities (1) and (6), it is possible to obtain the following dynamic equation of the time path of the unemployment rate:

$u_{t+1} = \lambda u_t + \Lambda u_t^2$ (7)

where $\lambda = (\alpha_D/\rho)$ and $\Lambda = (\alpha_I/\rho)$ (8)

If we use a suitable linear transformation of variable u_t in (7), we can write this equation in an equivalent way that leads to the corresponding "logistic map" (May 1976).

The transformation is $u_t = -(\lambda/\Lambda)U_t$ (9)

The logistic map is $U_{t+1} = \lambda U_t(1 - U_t)$ (10)

The coefficient λ is the characteristic parameter of the map itself, and equation (10) is the equation of the time path of the supporting variable U_t.

4. Graphical representations and evidence from the applications of the model

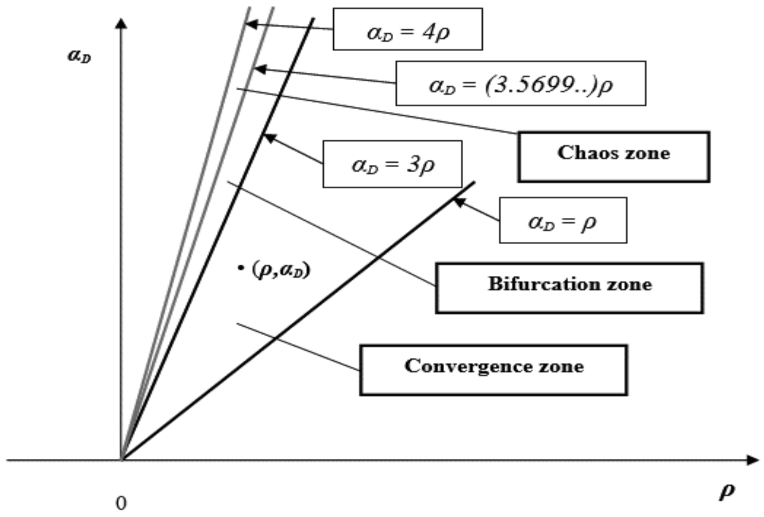
The analysis of the possible time paths of variable U_t (and consequently of the unemployment rate u_t) in relation to the variability of the characteristic parameter λ leads to the conditions for the existence of steady states,

[6] In the Appendix, we analytically develop a theoretical proof of dependence of the GDP growth rate gap on the square of u_t.

bifurcations, and periodic cycles and to the values of the same parameter related to the onset of deterministic chaos.

The numerical and analytical consequences of the variability of parameters ρ and α_D are visualized in Figure 1.

Figure 1. Sets of pairs (ρ, α_D).



The four half-lines in Figure 1 correspond to the cases related to the following equalities: $\alpha_D = \rho$ (half-line a); $\alpha_D = 3\rho$ (half-line b); $\alpha_D = (3.5699 \ldots)\rho$ (half-line c); $\alpha_D = 4\rho$ (half-line d).

These half-lines delimit three regions - in the first quarter - named “Convergence zone”, “Bifurcation zone”, and “Chaos zone”.

If (ρ, α_D) belongs to the open region named “Convergence zone”, then there is a potential convergence to the stable steady state $u_2^* = (\rho - \alpha_D)/\alpha_I$, which is nontrivial.

If (ρ, α_D) belongs to the open region named “Bifurcation zone”, then there are bifurcations and cycles characterized by increasing periodicity if ρ increases.

If (ρ, α_D) belongs to the open region named “Chaos zone”, then there is deterministic chaos.

If (ρ, α_D) belongs to half-line b, then there is the first bifurcation; if (ρ, α_D) belongs to half-line c, then there is the onset of deterministic chaos, which also persists if (ρ, α_D) belongs to half-line d.

For interesting economic interpretations and possible applications of these theoretical results, it is useful to observe that the values of parameters (or coefficients) ρ and α_D are influenced by important microeconomic aspects. The former expresses the ratio between the absolute value of the GDP growth rate gap and the contemporaneous unemployment rate, while the latter is the coefficient of the linear component in the quadratic function that expresses the relation between the GDP growth rate gap at time $(t+1)$ and the unemployment rate at time t. The former can be related to labor productivity, and the latter can be considered an indicator of rigidity of the labor market.

After analyzing our theoretical nonlinear model and discussing the stability of its dynamic solutions, we propose some applications to contemporary advanced countries to study the levels of complexity and of intrinsic disorder that characterize their unemployment dynamics. Moreover, in this paper we will use the results of these applications to express some forecasts of the deterministic nature of unemployment dynamics in those countries, the evolutions of which will look interesting.

4.1 Results for G7 countries: 1981-2017

We have applied the above analysis using data from the October 2018 IMF World Economic Outlook database for the unemployment rates, the values of GDP at constant prices and the output gaps of the considered countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States) for the period of 1981-2017[7]. The absolute value of the gap ($|g - g^e|$) for each country is calculated using the GDP values and their gaps expressed as percentages of potential GDP. Moreover, using the time series of this gap and of the unemployment rate u, we have determined, for each country, the values ρ, α_D and α_I as the coefficients of hypotheses H1, H2 and H5, respectively, using best-fit functions. The calculation of the values, for each country, of the characteristic parameter of the logistic map λ is simply based on the first of formulas (8). To calculate the coefficient in equation (1), we have estimated a first-degree interpolating function for the values of u_t and $|g_{t+1} - g^e_{t+1}|$, as suggested by the model, without a constant term. For the coefficients in (6), the value of u_t is considered in relation to $|g_{t+1} - g^e_{t+1}|$ by using a second-degree interpolating function, without a constant term.

[7] In our estimates, the year *t* in equations (1) and (6) is 1981.

In our numerical experiments, for the application of the logistic map we assume that the initial time ($t = 0$) coincides with the year 1981. It is possible to obtain the initial values of the supporting variable U_t. We consider transformation (9) in the initial time:

$$U_0 = -(\Lambda/\lambda)u_0 = -(\alpha_I/\alpha_D)u_0 \tag{11}$$

Aiming to determine the value of the unemployment rate to which each system tends or the range and its time series representation, it is necessary to calculate the values of u after determining the values of the supporting variable U obtained by the logistic map. The transformation is expressed by the following formula:

$$u_t = -(\lambda/\Lambda)U_t = -(\alpha_D/\alpha_I)U_t$$

In Table 1, we show the results of our model applied to the G7 countries for the period 1981-2017. The model parameters are followed by the value of u^* that represents the percentage (or the range) of the unemployment rate to which each system tends, given the parameters and the value of the unemployment rate in the initial year 1981.

Table 1. Values of the characteristic parameter of the logistic map for unemployment dynamics in the G7 group and the level of the trend of the unemployment rate (1981-2017).

Country	Coefficient α_D from function (6)	Coefficient ρ from function (1)	Coefficient $\lambda = (\alpha_D/\rho)$	u^* or fluctuation range
Canada	0.0029	0.0016	1.8125	6.50
France	0.0036	0.0009	4.0000	0.0002-12.00
Germany	0.0035	0.0015	2.3333	10.00
Italy	0.0039	0.0010	3.9000	1.24-12.68
Japan	0.0069	0.0033	2.0909	4.00
UK	0.0021	0.0012	1.7500	9.00
US	0.0027	0.0020	1.3500	7.00

Authors' analysis of the IMF data.
Note: the minimum and maximum values for France and Italy are obtained considering the first one thousand repetitions of the logistic map.

From the results of the application of our model (see Table 1), we deduce the following statement: the nonlinear dynamic of unemployment is sufficiently distant from the risk of onset of deterministic chaos that, however, is not to be excluded. In fact, in relation to G7 countries during the period 1981-2017, it is possible to observe that the values of the characteristic parameter $\lambda = (\alpha_D/\rho)$ mostly belong to the interval of real numbers (1,3), except for the chaotic circumstances of France and Italy.

Moreover, it is possible to observe from Table 1, consistent with the contribution of Moosa (1997) for the G7 countries, the difference between the values of coefficient λ of the US and Canada compared to values of European countries. This comparison can be related to different levels of flexibility of labor markets (see the finding of Lee 2000; Sögner and Stiassny 2002) and consequently to different levels of ability to adapt to the dynamics of the economic system, which is relatively stronger in Canada, the UK and the US. These results are consistent with the OECD indicator of Employment Protection Legislation "Protection of permanent workers against individual and collective dismissals". This indicator shows a value higher than 2 for all countries[8] except Canada (1.51), the UK (1.59) and the US (1.17).

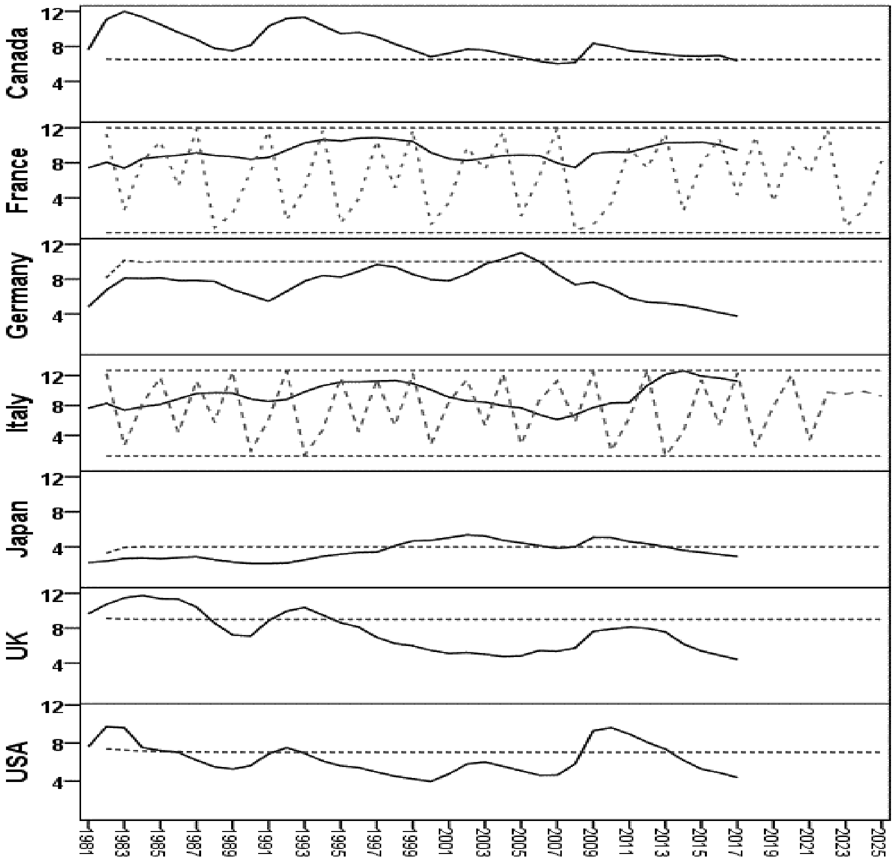
The results suggest that the considered advanced economies have several possible time paths of unemployment. In view of the economic literature, the unemployment rate dynamics is strongly linked to institutional and structural characteristics of each country. The greater adaptability of the labor market to short-term adjustments, e.g., in the US, has sustained a rapid economic recovery after the 2007 crisis and enabled the control of the employment dynamics. In contrast, the rigidity of labor legislation makes it difficult to adjust employment in different economic circumstances, e.g., in Italy, leading in turn to the risk of unpredictability and especially to high unemployment levels in periods of economic slowdown.

Applying transformation (9) to the results of this numerical experiment based on the logistic map, it is possible to forecast approximately - using the interval of the considered iterations - the ranges of variability of the unemployment rate in the seven countries until 2025. This type of forecast reflects only the intrinsic and deterministic dynamics of the system that originate from the systemic characteristics in the considered initial year, without taking into account random exogenous events and policy

[8] France: 2.82; Germany: 2.84; Italy: 2.89; Japan: 2.09. The available OECD data are for 2013.

interventions. Figure 2 shows the graphs for the seven countries, representing the time series of the unemployment rate, the results from the logistic maps and some forecasts.

Figure 2. Unemployment rate (black lines, %), u^* (gray dotted lines) and ranges[9] for the G7 countries (1981-2017).



Source: Authors' analysis of the IMF data

The 1981-2017 period that we have considered for applying the main results of our model in the G7 countries includes the international economic crisis of 2007. In Figure 2, we observe that the inclusion of the effects of the

[9] For France and Italy, both the range of values (horizontal lines corresponding to minimum and maximum values based on one thousand repetitions) and the values of the logistic map are reported.

crisis in the analysis and the review of the obtained results support the possibility of applying the model to contexts with severe fluctuations. In view of these observations it is interesting to consider the G7 countries in the period of recent Great Recession (Section 4.2).

4.2 Preceding two decades and the Great Recession

In the following analysis, we apply the logistic map to the G7 countries considering the period 2000-2017. This period should contain sufficient observations to verify the Okun's relationship; at the same time, approximately half of the years are characterized by recessionary effects in many countries. We show forecasts in this case also to observe differences with respect to the previous analysis. We expect that the major economies experience fewer exogenous effects and therefore exhibit a more evident presence of Okun's Law. This should mean that at least the US, Japan and Germany (the world's 1st, 3rd and 4th economies, respectively, according to the GDP at current prices of 2017) have values of λ similar to that estimated in Section 4.1.

Table 2. Values of the characteristic parameter of the logistic map for unemployment dynamics in the G7 group and the level of the trend of the unemployment rate (2000-2017).

Country	Coefficient α_D from function (6)	Coefficient ρ from function (1)	Coefficient $\lambda = (\alpha_D/\rho)$	u^* *or fluctuation range*
Canada	0.0055	0.0015	3.6667	2.57-8.40
France	0.0081	0.0008	10.1250	-
Germany	0.0025	0.0018	1.3889	7.78
Italy	0.0046	0.0012	3.8333	2.35-14.69
Japan	0.0072	0.0030	2.4000	4.20
UK	0.0039	0.0014	2.7857	6.25
US	0.0046	0.0018	2.5556	7.00

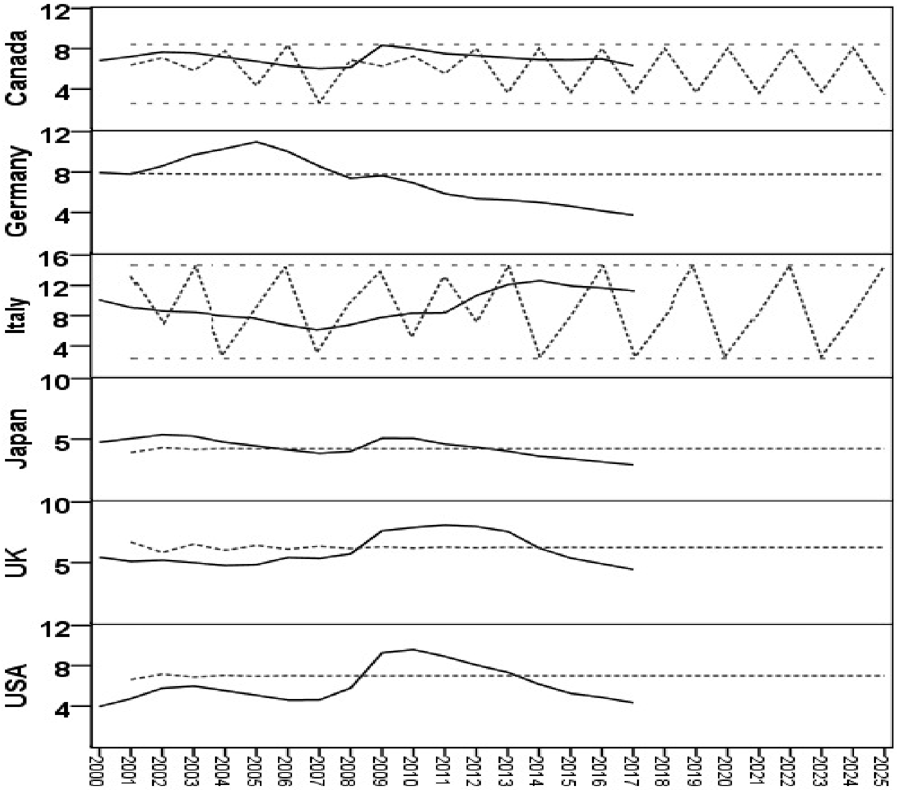
Authors' analysis of the IMF data.
Note: the minimum and maximum values for Canada and Italy are obtained considering the first one thousand repetitions of the logistic map.

In Table 2, four countries out of seven are in the "Convergence zone"; Canada is in the "Chaos zone", as is Italy (confirming the result for this country in Table 1), while the λ value of France exceeds the threshold value of 4 and does not allow making estimates for the period considered. We observe evident effects of the recent crisis in the values of λ and u^*. In Canada, a notable increase in λ is evident, but the value of the unemployment trend level previously estimated is within the range estimated for 2000-2017. The trend values of Germany and the UK have decreased despite the crisis, due to the rapid economic recovery, while the Italian range has widened both in amplitude and in the absolute values of the level of unemployment. Japan's value of u^* increased by only 0.2%, and, as expected, the respective value of the largest global economy - the United States - remained unchanged, suggesting an unemployment level of 7%.These changes confirm that the dynamic of unemployment is characterized by complexity because it depends on the initial conditions, *i.e.,* on the initial year of the analysis. This observation leads us to be sure of the significance of nonlinearity in the search for a more fitting quantitative relation between the unemployment rate and the growth rate gap of the GDP.

In Figure 3, we show the time series, the results from the logistic map and the 2018-2025 forecast of the unemployment rate[10].

[10] The graph for France has been omitted because the model is inapplicable for $\lambda > 4$.

Figure 3. Unemployment rate (black lines, %), u^* (gray dotted lines) and ranges[11] for the G7 countries (2000-2017).



Source: Authors' analysis of the IMF data

A detailed examination of the crisis years shows that the most recent time series represented in Figure 3 are consistent with the results of the logistic map - values to which each system would tend and particularly their "predictability" - despite the trends varying by country. An increase in unemployment of up to 5 percentage points occurred between 2007 and 2009 in all countries except Germany. In the more resilient countries, an inversion of the trend emerged since 2010 (Canada, Germany, and Japan), 2011 (the US) and 2012 (the UK), while in Italy the first decrease in u occurred in 2015.

[11] For Canada and Italy, both the range of values (horizontal lines based on the minimum and maximum values based on one thousand repetitions) and the values of the logistic map are reported.

These observations are a strong motivation to revisit Okun's Law. Our results are consistent, for example, with the findings of Lee (2000) that shows that the relation is less robust in OECD developed countries, while Zanin and Marra (2012) prove that Okun's relationship has varied in the preceding decade and that it is spatially heterogeneous and time-varying in Europe. Furthermore, Malley and Molana (2008) investigate the implications of the different algebraic signs of the relation between output and unemployment due to rigidities or distortions, observing that only the data for Germany, among the G7 countries, support a monotonic relation.

5. Conclusions

The empirical evidence involved in Okun's Law is strongly confirmed by many studies, but the linearity of the relation is not universally true, and it is not very informative about deeper aspects of dynamics of unemployment in contemporary developed countries. In addition, recent studies suggest that in periods of crisis, the unemployment rate is considered an effective summary measure of the economic conditions (e.g., Daly *et al*. 2014).

In this paper, we test the significance of nonlinearity in unemployment dynamics using tools of the mathematics of chaos and simultaneously introducing a formalization of an indirect quadratic effect on the unemployment rate of the next period, deriving from a chain reaction conveyed by the aggregate demand, resulting from the primary effect on unemployment itself. It is well known that nonlinearity may imply chaos, fluctuations, instability and, under certain conditions, convergence to stable steady states. Of course, these results imply the possibility of adjusting policy interventions towards desirable employment targets. Our analysis shows that the complexity of the aggregate global system will determine all these dynamic aspects of unemployment's evolution over time.

Both the diverse countries in the G7 group and the recessionary years were useful for applying the model. The average annual variation of the pre-crisis decade shows a negative sign for all countries, while the countries that show a risk of chaos in the unemployment dynamics − Canada, France and Italy − are the only ones to have a positive variation of the post-crisis average annual unemployment rate (2008-2017)[12]. All the other countries showed a rapid recovery towards pre-crisis values.

[12] The average increase is 0.03% in Canada, 0.15% in France, and 0.51% in Italy (obtained by analysis of the IMF data).

Of course, the local labor market institutions are the direct determinants of the differences in the effects on the involved variables, particularly unemployment, which is consistent with the economic literature (Stockhammer and Klar 2011) and with the interpretation of our results, as shown in the application of our model to the G7 countries. In our results, differences between European and North American countries indeed emerge, thus confirming that flexibility of the labor market, which is revealed to be at different levels in the two groups of countries, is an important determinant of unemployment dynamics.

However, numerical disorder in unemployment dynamics is possible. In fact, we observe the cases of France, Italy and partially Canada, in which unemployment dynamics evolve towards disorder and uncontrollability if the challenging period of the Great Recession is considered.

The analysis contained in this paper suggests the possibility of controlling unemployment dynamics by two coefficients (α_D and ρ), the ratio of which must not exceed a specific threshold value to avoid chaos. The former can be considered an indicator of rigidity of the labor market, while the latter can be related to labor productivity. Our study confirms that these two aspects are crucial for the control of unemployment dynamics and for the achievement of stable results (for the role of labor market flexibility, see the contributions of Guisinger *et al.* 2018 and of Oh 2018).

In the search of cases at risk for chaos, our analysis shows, e.g., that in Italy, the endogenous weakness in potential economic development generates instability and uncontrollability of unemployment, which is due both to the presence of a rigid labor legislation and to the inadequacy of labor productivity. Indeed, the characteristic parameter of the logistic map depends positively on the increase in the rigidity of the labor market and negatively on the increase in labor productivity. In Italy, as the labor market is relatively rigid and the labor productivity is under its potential level[13], the parameter of the logistic map is higher than the threshold value, determining chaotic time paths of the unemployment rate. This disorder in the dynamics of the unemployment rate is also related to the lack of economic recovery after the 2007 crisis in this country, which is evident from the considered time series data. The results of Italy confirm the very basis of Okun's intuition, which

[13] In Italy, over 20 years of legislative actions to increase flexibility induced a block in labor productivity growth (Lucidi and Kleinknecht 2010).

highlights the role of an appropriate economic growth rate in solving the socioeconomic problems related to unemployment. Thus, the findings of our work sustain Okun's idea by adopting a nonlinear dynamic approach and complete it by criteria for the control of instability and economic disorder.

The bifurcations and chaos depend on the values of the characteristic parameter of the logistic map and, therefore, on the two coefficients that we have described above. From the microeconomic point of view, considering the dynamic hypotheses that introduce these coefficients, it is interesting to observe that the first of them embeds the effects of the behavior of households in relation to consumption spending and the effects of investment choices of firms. The second coefficient expresses the technological links that exist between the variables.

These observations allow us to assert that our contribution is a revision of Okun's Law that consists not only of the consideration of nonlinearity but also of a novel interpretation of the microeconomic causes of the relationship between unemployment variation and the GDP gap over time. Finally, we observe that ability of our analysis to forecast the ranges of variability of the unemployment rate is not due to the use of techniques belonging to expectation modeling or forecast modeling. Our model is based on theoretical dynamic relations (expressed by the hypotheses of Section 3) that involve intrinsic aspects of the system. These intrinsic aspects are embedded in the two coefficients (and their ratio) that we have used in our model. Our characterization of their macroeconomic and microeconomic interpretation provides new suggestions for a deeper revision of Okun's Law and for policy interventions.

Appendix

Analytical micro-founded macro model supporting the dependence of GDP growth rate gap on the square of u_t

Assume that output is produced according to the following production function:

$$Y_t = A_t\,[(1 - u_t)L_t\,]^{\alpha}\,K_t^{\beta} \qquad (\alpha > 0, \beta > 0) \tag{1.A}$$

where A_t is the total productivity of factors, L_t is the labor force (measured by the total number of hours worked), u_t is the unemployment rate, and K_t is the aggregate stock of physical capital. If N is the total number of employees (constant over time) and l_t is the average number of hours worked per employee, then $L_t = l_t N$.

Equation (1.A) is equivalent to the following:

$$Y_t = \Phi_t\,(1 - u_t)^{\alpha} \tag{2.A}$$

where $\Phi_t = \Phi_t(A_t\,, l_t\,, K_t) = A_t\,L_t^{\alpha}\,K_t^{\beta}$.

We focus on business cycle fluctuations and assume a constant level of potential output (for simplicity, we assume it to be equal to zero). Furthermore, we neglect short-term variations in physical capital. In this case, variations in Φ_t reflect short-term variations in productivity A_t and short-term variations in the intensive margin of employment l_t. Suppose that we take the process $\{\Phi_t\}$ (*i.e.,* productivity and hours worked per employee) as a given. This assumption is only made for simplicity and can, of course, be weakened. However, the search-and-match framework uses this assumption.

Using $Y_t = (1 - u_t)^{\alpha}\,\Phi_t(A_t\,, l_t\,, K_t)$, we obtain

$$\ln Y_{t+1} = \alpha\,\ln(1 - u_{t+1}) + \ln\Phi_{t+1}(A_{t+1}\,, l_{t+1}\,, K_{t+1})$$

Using the approximation $\ln(1\text{-}\ z) \approx z$, we obtain

$$\ln Y_{t+1} \approx \alpha\,u_{t+1} + \ln\Phi_{t+1}(A_{t+1}\,, l_{t+1}\,, K_{t+1}) \tag{3.A}$$

Taking the process $\{\Phi_t\}$ as a given, any macro model of the labor market will determine an equilibrium law of motion that can be generically expressed by the following map:

$$u_{t+1} = \mathrm{G}(X_t, \Phi_{t+1}, \Phi_t), \qquad (4.A)$$

where X_t is the relevant endogenous state variable. The definition of X depends on the model at hand; for our purpose, we let $X_t = u_t$ and thus, from (3.A) and (4.A), we obtain

$$\ln Y_{t+1} \approx \alpha\, \mathrm{G}(u_t, \Phi_{t+1}, \Phi_t) + \ln\Phi_{t+1}(A_{t+1}, l_{t+1}, K_{t+1}).$$

If G is a map separable in u and Φ, it is possible to obtain

$$\ln Y_{t+1} \approx \alpha \mathrm{G}_1(u_t) + \alpha \mathrm{G}_2(\Phi_{t+1}) + \alpha \mathrm{G}_3(\Phi_t) + \ln\Phi_{t+1}(A_{t+1}, l_{t+1}, K_{t+1}). \qquad (5.A)$$

Under the standard hypotheses of derivability with respect to variable t, we can calculate the derivative D_t of both sides of equation (5.A) and obtain

$$\mathrm{D}_t(\ln Y_{t+1}) \approx \alpha\, \mathrm{D}_t[\mathrm{G}_1(u_t)] + \alpha\, \mathrm{D}_t[\mathrm{G}_2(\Phi_{t+1})] + \alpha\, \mathrm{D}_t[\mathrm{G}_3(\Phi_t)] + \mathrm{D}_t[\ln\Phi_{t+1}(A_{t+1}, l_{t+1}, K_{t+1})].$$

Moreover, as $\mathrm{D}_t(\ln Y_{t+1})$ is equal to the growth rate at time $(t+1)$, and according to the variability of the relevant endogenous state variable only, we obtain $g_{t+1} = \alpha\, \mathrm{D}_t[\mathrm{G}_1(u_t)]$, which we transcribe as follows, using the definition $\mathrm{F}(u_t) = D_t[\mathrm{G}_1(u_t)]$:

$$g_{t+1} = \alpha \mathrm{F}(u_t)$$

Using a linear-quadratic approximation of $\mathrm{F}(u_t)$, we obtain

$$g_{t+1} = \alpha\,(\beta_1 u_t + \beta_2 u_t^2) = (\alpha\beta_1)u_t + (\alpha\beta_2)u_t^2 \qquad (\beta_1 \neq 0.\ \beta_2 \neq 0).$$

Finally, with $\alpha_D = \alpha\beta_1$ and $\alpha_I = \alpha\beta_2$, we can write

$$g_{t+1} = \alpha_D u_t + \alpha_I u_t^2,$$

which matches equation (6) of this paper if $g^e_{t+1} = 0$.

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