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THE SYSTEMIC THINKING OF BRUNO DE FINETTI

Abstract

The *mechanistic* scientific approach, based on principles such as that for which the microscopic world is simpler than the macroscopic one, and that the macroscopic one can be expressed through the strict knowledge of infinite details, it has been challenged by many questions, one of which is represented by the so-called “The Three-Body Problem” (Barrow-Green, J., 1996).

New cognitive approaches have been introduced based, just to name a few, on the theoretical role of the observer, on non-linearity, on principles of uncertainty, on constructivism, on the systemic point of view and on emergence.

Here, I wish to emphasize how the treatment of uncertainty, developed according to Bruno de Finetti’s vision, fits perfectly in these types of approaches, rather he anticipates them, stopping to highlight what is the essential role of one who makes probability assessments, which we will call the *observer*.

Unlike objectivist orientations, in particular those that refer to a purely statistical view of the reality, Bruno de Finetti had, in the last century, the fundamental intuition that the probability assessments of an event do not represent anything else if not *the degree of believe* who a *coherent individual*

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has in realizing that event. This degree of believe is determined on the basis of the *information* that the individual possesses regarding that event and in this sense the individual plays the role of observer, as happens in the modern *theory of complexity*.

de Finetti taught us that the logic of uncertainty explores the context of the possible, accepting the condition of being unsuitable for making *predictions*, on the contrary providing the tools to assign probabilities to events, so as to be able to make *previsions* and take decisions.

Traditionally, the concept of probability has been considered as a consequence of our *ignorance*, of our limitations. In the same way, the uncertainty's principles were considered by mechanistic thinking as a limitation of our knowledge of infinite details.

Now we can instead consider the probability as our *tool* to describe the nature.

The methodology followed by de Finetti is based on the *coherence* in the assignment of probabilities to single event and families of single events.

The coherence is the essential tool that allows us to make probability assessments and, in particular, to update the assessment of an event when a new (or supposed) information becomes usable (of interest).

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We desire *truth*, and find within ourselves only *uncertainty*

Blaise Pascal, *Thoughts*.
The Harvard Classics. 1909–14.

It would therefore seem natural that the customary modes of thinking, reasoning and deciding should hinge explicitly and systematically on the factor *uncertainty* as the conceptually pre-eminent and determinative element.

Bruno de Finetti, *Theory of Probability: A Critical Introductory Treatment*, 2 volumes (Translated by A. Machi and A. Smith). Wiley, London (1974).

1. Coherent probabilities assignments and emergence

I propose to show how the problem of a *coherent* assignment of probabilities to a family of event, by an *observer*, is related to *emergence*, in the way it is considered in the modern science.

I remember that *emergence* (Corning 2002, Minati and Pessa 2002, Pessa 1998, 2002), in this context, may be considered as a process of formation of new *collective entities* (which require a level description different from that used for the single elements), as swarms, flocks, traffic, industrial districts, markets; *collective effects*, such as super conductivity, iron-magnetism, laser effect. These entities are determined by *coherent* behavior (as perceived by an observer) of interacting components.

Referring to the cognitive model of (Anderson 1983) used by the observer, emergence may be seen as a process depending only by the observer, considering that collective properties *emerge* at a higher level (ie more abstract) than to the one he used to treat the components. Collective properties are detected by the observer as new, depending on the cognitive model assumed, suitable for establishing the presence of coherence.

But more, the subjective conception of de Finetti's probability is all built as a *logically open cognitive system*. In fact, the observer (the evaluator) plays an *active* role, sensitive to the *context*, who moves in a *non-objectivistic context*, is *process-oriented*, *learns dynamically* from experience in an *inductive* way, is an *integrating part of the system* and *generates its existence*, finally he can *choose the rules* to use (Minati 2004).

I will start from this last point to illustrate what has been said. In assigning a measure of uncertainty to a family of events, of which we do not know the truth value, the observer may follow two paths, depending on the context in which he finds himself (or to which he chooses to refer).

The first consists in introducing a *natural order* among the events: if we suppose, as it is legitimate, that an event compared with another may be or *more possible* or *less possible* or *equally possible* than this, he can easily construct a *relation of order of not less possible* within the family of events considered.

This relation has the following properties:

An uncertain event E_h of the family, is always more possible than the impossible event and less possible than the certain event and is no less possible than itself.

If an event E_h is no less possible than E_k , then E_k cannot be no less possible than E_h , unless E_h and E_k are equally possible.

If an event E_h is no less possible than E_k and E_k of E_j , then E_h is no less possible than E_j .

If an event E is incompatible with E_k and with E_j and E_k is no less possible than E_j , then the union of E with E_k is no less possible than the union of E with E_j .

In this way, the observer introduces a *qualitative measure* of the uncertainty of an event.

It should be noted, in passing, that in the particular case of dealing with a certain event partition in cases judged to be equally possible, it follows that the introduced qualitative order immediately translates in a quantitative measure of their uncertainty as a ratio between the number of favorable cases and the number of possible cases.

If, then, a further property related to conditional events is considered:

The events E_h and E_k imply E , then E_h/E is not less possible than E_k/E if E_h is not less possible than E_k ;

Its introduction together with the previous, allows to *qualitatively* develop the whole theory of probability (de Finetti 1937).

The second way is that for which the observer explains his own *degree of believe* in occurrence of an event by means a quantitative assessment.

To do so, it needs a measuring *tool* that is suitable for the purpose. Bruno de Finetti proposed two equivalent measurement criteria: that of the *bet* and that of the *penalty* (de Finetti 1974).

I recall here only that relating to bet, which appears more natural since it reflects historically what happened in the construction of the probability theory.

Suppose we have to bet a certain amount to win another one when a determined event occurs. This is what common happens in all betting fields.

Then make a *bet* on an event E means that you are willing to pay a *part* of a certain sum S (>0), which we can indicate with pS , to receive S if E occurs and 0 in the opposite case.

If we introduce the G_E *gain* function with respect to that given bet, we will get the following schematic:

$$G_E = \begin{cases} S-pS, & \text{if } E \text{ occurs} \\ -pS, & \text{if } E \text{ does not occur.} \end{cases}$$

It should be noted that it is quite clear that in a bet that each of us has the goal of maximizing our own gain, so this fact could lead us to distort our assessment; how do defend ourselves from this to prevent it bet tool becomes arbitrary and therefore ineffective for the purpose to which it must respond?

Meanwhile, it must be ensured that Those who bets pS to receive S (from the other bettor) if E occurs, must likewise be willing to pay S to receive pS if E occurs, ie to exchange the terms of the bet with the other bettor, this will ensure that the assessment made by the individual reflects his or he degree of believe without being influenced by the desire to make a major gain, which otherwise could be achieve by the other. For example, suppose that in a bet on E , I think you can pay 70 to receive 100 in case of E occurs, I might think about increasing my hypothetical gain (in this case, equal to 30), declaring to be willing to pay 40 ; but if I am willing to exchange bet odds with the other bettor then my hypothetical gain could significantly decrease (-60)!

But this is not enough, it must ensure that the possible values of the G_E gain are not both of the same sign, because only in that case would there be a certain win or loss, regardless of the occurrence or not of E . With the consequence that only in such a case an individual would accept to bet. This condition was appropriately called by de Finetti *coherence*.

The coherence in a bet on an event E as it is well known establish that, given S equal to 1 , (but it is also valid for $S \neq 1$) in any case the price p that an individual is willing to pay to receive 1 if E occurs is always between 0 and 1 . Moreover, the same coherence imposes that if E is *certain*, then p must necessarily be 1 and if E is *impossible*, then must necessarily be 0 .

It should be noted, however, immediately that $p=1$ not implies E *certain*, nor does $p=0$ implice E *impossible* (for further details see de Finetti 1974).

So, following de Finetti, the probability of an event E (or the the *numerical measure* of uncertainty on E) is the price p (real number) that an individual is willing to pay in a *coherent bet* to receive 1 if E occurs and 0 otherwise.

But just a bet will be coherent only if the *observer* (the individual) who evaluates the uncertainty of E make it so.

There will therefore exist, whith respect to E infinite coherent assessments, provided they are between 0 and 1 !

How then will the observer choose one? He will have to rely on the *information* he has about E and express this information through a number.

Naturally, the more the information will be *rich*, the more the individual will be less doubtful in choosing one among infinite numbers!

Now in measuring information, an individual, as always happens, does not have an *objective* evaluation criterion: in fact, personal beliefs, feelings and all those characteristics that contribute to the formation of a judgement will intervene. So, he will only have to formulate, we could say so, objectively how much he assesses subjectively!

This is how de Finetti expresses in *L'invenzione della verità* (2006):

“Any behaviors...in sense of believing that the occurrence of this or that event does is plausible therefore depends only on a feeling, on that same feeling that must honestly present as the true starting point, and that someone prefers instead to ignore and modestly hide behind a barricade of logical devices as cumbersome as they are empty”.

Obviously, in some cases, it will be much easier: for example, if an individual judges equally possible the occurrence of E and that of its negation E^c , to both will assign the value of probability 1/2 (but are we sure that judge equally possible two *real* events would be so simple and natural?). While, if He judges five times more probable E compared to E^c , then it will assign to E probability 5/6 and to E^c 1/6 .

In other cases it may resort to assessments based on the relative frequency, but only, as de Finetti has well specified, when the event E is part of a family of *exchangeable* events, that is when the assessment of probability of any n-tuple of family events considered depends only on the number of fixed events and not of particular fixed events; in short: it depends only on *how many* events and not *which* they are considered (de Finetti 1974)!

For example, in the extraction with return of balls from an urn of unknown composition, that is, for which we know the total number of balls but not the percentage of *red* ones, an individual who want to evaluate the probability to get a *red* ball at n-th extraction, having been extracted (n-1)/3 red on (n-1), it could estimate equal to (n-1)/3 the probability that the red ball comes out at the n-th extraction, because the events of the family considered are exchangeable (we only care *how many* red balls are extracted). But if we would like to evaluate the probability of a certain boxer victory at the 101-th match of his career, knowing that he won 85 on previous 100, would that be enough to estimate this probability equal to 85/100? Obviously no, because it could, in the worst case, have lost all the last 15 matches, and the degree of believe in him... may not be so high!

1.1. The case of random numbers and complex phenomena

I have done so far considerations relating to single events or to families of analogous events. Often however, we run into random phenomena, which in some cases can be described by *random numbers*. Such as the X number of fatal car accidents in a year relating to people who did not wear a seat belt, for which one can consider events of the type ($X = n$). Or in more complex phenomena, with respect to which we may be interested in such events: the amount of rainfall in a given Italian city in the next year will be greater than that of the past year, or the average humidity will be lower, or the measured smog level will be higher, or fuel consumption will be equal. In the first case, that of random numbers, it is possible to formulate various probabilistic models that allow us to evaluate the probability of any event related to them, but we must not forget that these evaluations are not objective, as they apparently may seem, because given the model is enough apply formulas to get them; vice versa it is always the observer who chooses, based on his information, the model he considers most suitable to describe the phenomenon considered. What does it happen, instead, in the second case?

Again, the systemic aspects of the approach adopted by de Finetti give an exemplar answer.

I remember that in the classical approach (commonly used in applications) relating to a random phenomenon we proceed as follows: first we define a Ω space of the results or elementary cases possible, or we construct a certain event partition, then we assign a probability to each of these cases (or constituents) and since any event related to that phenomenon can be obtained as a union of constituents, a probability is attributed to each of them in a linear manner. I note that of course the problem of how to assign probabilities to constituents remains open!

On the contrary, de Finetti bases his conception on the fact that every event is single and for each of them we can express our degree of believe through a qualitative or quantitative evaluation. If we then find ourselves in front of a single event and have evaluated its probability, we need to assign probabilities to further events how should we behave?

Several cases may arise. If you are dealing with a family of events E_i that forms a partition of the certain event, then for coherence the sum of the probabilities of the single E_i must be equal to 1 and the probability of the union of n incompatible events must be equal to the sum of the single probabilities.

If, again, there are n events and a coherent probability assignment, then the probability of an event that depends linearly on the first n is determined.

In the cases, instead, in which between the events E_{ii} considered and a new event E there is a logical connection, then always for the coherence it must proceed in this way: we build the constituents relatively to the given family (that is all the intersections possible between events so that in each appears one of them or its contrary, e.g. $E_1 \cap E_2 \cap \dots \cap E_h \cap E_{h+1}^c \cap \dots \cap E_{n-1}^c \cap E_n^c$), then two events are identified E' and E'' , which are respectively the maximum event of all the constituents implying E and the minimum event of all the constituents implied by E : necessarily the probability of E must be within the closed interval $[P(E'), P(E'')]$.

Even in this situation, therefore, the probability of E is not univocally determined, it can simply shrink the interval $[0,1]$ within which the observer can evaluate it to be coherent. Then having indicated with E' and E'' , two events which are respectively the maximum event union of all the constituents which implies the event E , and the minimum event union of all the constituents which are implicated by the event E : the probability of E may be included necessarily in the closed range $[P(E'), P(E'')]$. Even in this situation, therefore, the probability of E is not univocally determined, it can simply shrink the interval $[0,1]$ within which the observer can evaluate it to be coherent.

But the most interesting situation occurs when dealing with individual events that are examined starting with the first and then the others afterwards. To each of these the observer, based on his own information, assigns a coherent probability value, i.e. between 0 and 1. In this case, however, there may be logical links between events that have not been considered or, otherwise, referred to information was received only when all the events were introduced, then how to check whether the assessment is overall coherent?

It will be necessary to construct the constituents starting from the events considered, and since each of these latter will result in the union of some of the constituents obtained, its probability must be equal to the sum of the probabilities (not yet determined, therefore let's say so unknown) of these constituents. In this way we will obtain a system of n equations in s ($\leq 2n$) unknown x_i , with the constraints $x_1 + x_2 + \dots + x_s = 1$ and $x_i \geq 0$. If there is a complete solution of the system then the evaluation given may be said to be coherent.

It is interesting to note that the system may not allow solutions, in which case the evaluation would be incoherent; that can admit a single solution, but that there can also be more solutions, that is a set of different evaluations all coherent. If exists an s-tuple solution of the system, the given assessment could be defined as coherent.

I will clarify this last interesting aspect through examples.

Given three events A, B, C and one observer would have evaluated their probabilities as follows $P(A)=1/2$, $P(B)=2/5$, $P(C)=1/5$ (obviously, each of them represents a coherent assessment!).

Let it be known then that $A B C = \Phi$ (with $A B C = A \cap B \cap C$).

So, the possible constituents are:

$$Q_1 = A^c B C, Q_2 = A B^c C, Q_3 = A B C^c, Q_4 = A B^c C^c, Q_5 = A^c B C^c, \\ Q_6 = A^c B^c C, Q_7 = A^c B^c C^c.$$

To establish then whether, under the given conditions, the overall evaluation expressed by $P(A)=1/2$, $P(B)=2/5$, $P(C)=1/5$ would result coherent, it is necessary to establish whether the following system admits at least one solution:

$$\begin{cases} x_2 + x_3 + x_4 = 1/2 \\ x_1 + x_3 + x_5 = 2/5 \\ x_1 + x_2 + x_6 = 1/5 \end{cases}$$

with $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$ and $x_i \geq 0$, $i = 1, 2, \dots, 7$.

If it is posed, as it is legitimate to do, $x_1 = 0$, $x_3 = 0$, $x_6 = 0$, with simple steps we get the following system solution

$$x_1 = 0, x_2 = 1/5, x_3 = 0, x_4 = 3/10, x_5 = 2/5, x_6 = 0, x_7 = 1/10;$$

therefore, the allocation of assigned probabilities to the events A, B, C determine a coherent overall assessment!

Note that if we put $x_2 = 0$, $x_3 = 0$, $x_6 = 0$, we would have obtained a different one system solution

$$x_1 = 1/5, x_2 = 0, x_3 = 0, x_4 = 1/2, x_5 = 1/5, x_6 = 0, x_7 = 1/10;$$

and also, in this case, the overall assessment would have been coherent!

Not only, but if initially the observer had evaluated $P(A)=\alpha$, $P(B)=\beta$, $P(C)=\gamma$, with the condition $A B C = \Phi$, we would have obtained the following system

$$\begin{cases} x_2 + x_3 + x_4 = \alpha \\ x_1 + x_3 + x_5 = \beta \\ x_1 + x_2 + x_6 = \gamma \end{cases}$$

with $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$; $x_i \geq 0$, $i = 1, 2, \dots, 7$,
and $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, $0 \leq \gamma \leq 1$.

With the choices of $x_1 = 0$, made in the two cases examined above, we would have had respectively the following solution:

$x_1 = 0, x_2 = \gamma, x_3 = 0, x_4 = \alpha - \gamma, x_5 = \beta, x_6 = 0, x_7 = 1 - (\alpha + \beta)$,
 which would give rise to a coherent overall assessment, as long as would be $\alpha \geq \gamma$ and $\alpha + \beta \leq 1$;

$x_1 = \gamma, x_2 = 0, x_3 = 0, x_4 = \alpha, x_5 = \beta - \gamma, x_6 = 0, x_7 = 1 - (\alpha + \beta)$,
 that would give place to a total coherent assessment, provided it is $\beta \geq \gamma$ and $\alpha + \beta \leq 1$.

2. Learning by experience

I now turn to examine the additional aspects of the initial assumption, in particular for as regards learning inductively by experience, referring to conditional events.

I recall that, given any two events E and H ($H \neq \Phi$), we can consider a new entity E/H, to which the name of conditional event is assigned, which has the following meaning:

E/H =	TRUE,	if H true and E true
	FALSE,	if H true and E false
	UNDETERMINED,	if H false

So, if we want to make a (conditional) bet relative to E/H we will, according to de Finetti¹, behave as follow:

PAY p	TO RECEIVE	1	if H true and E true
		0	if H true and E false
		p	if H false

From this definition of conditional bet it follows that we can measure the uncertainty of the E/H event through the price p that a coherent individual is willing to pay with the previous outcomes and this measure is called conditional probability and denoted P (E/H).

de Finetti proves, moreover, how the conditional probability, thus introduced, verifies all the properties of a probability and, finally, as the only possible reading of P (E H) is that of the probability of E supposed to be true H.

¹Bruno de Finetti, *Theory of Probability: A Critical Introductory Treatment*, 2 volumes (Translated by A. Machi and A. Smith). Wiley, London (1974).

In particular, he shows how the simple (and natural) condition of coherence: the unevenly negative random gain in a set of bets, leads to the *multiplication theorem of probability*:

given two events, any E and H ($H \neq \Phi$) we have

$$P(E \cap H) = P(H) P(E/H);$$

and its corollary, the *Bayes theorem* (also with $E \neq \Phi$):

$$P(H/E) = K P(H) P(E/H), \text{ with } K = 1/P(E), P(E) \neq 0,$$

which tells us that the conditional probability of H given E is equal to the product of the unconditional probability of H for the conditional probability of E given H, within a proportionality's not null K factor.

Let us now focus on the meaning of Bayes' theorem. In reality, it tells us much more if we give a reading of this kind: let it be an event that represents the outcome of an experiment of a given random phenomenon and let H be a hypothesis concerning the same phenomenon, then the theorem states that the conditional probability of the hypothesis H given E is proportional to the unconditional probability of the hypothesis H multiplied for the conditional probability of E given H.

To make it clearer what we have expressed, we use the classic example of the urn of *unknown composition*. In other words, let us consider an urn of which we know that it contains N balls, but of which the percentage of red balls is not known (being able, therefore, to be present in the urn from 0 to N red balls). The event E represents a possible result related to the extraction of n balls from the urn (for example, without restitution): h red balls on n, and event H is the hypothesis: r balls are present in the urn red on N. Then the Bayes' theorem let us to evaluate the conditional probability of the hypothesis H given the event E (we may call it *final probability*), through the unconditional probability of H (we may call it *initial probability*) and the conditional probability of E/H, such as of E supposed as true H (it may be calculated easily and we may call *likelihood*), always less than the proportionality factor K.

Ultimately, Bayes theorem tells us how we need to update our probability assessment in the presence of new information (or rather, suppose that new information becomes known to us):

$$\textit{final probability} = K \times \textit{initial probability} \times \textit{likelihood}.$$

Observe that the initial and final have, in this context, the only meaning respectively of before and after it becomes known (suppose we know) E. Naturally, in the same way the final probability of hypothesis H^c can be evaluated, resulting

$P(H^c/E) = K P(H^c) P(E/H^c)$, with $K = 1/P(E)$, $P(E) \neq 0$.

And, more generally, if we want to evaluate the final probability of m different hypotheses H_j , forming a certain event partition, we have the following expression of Bayes theorem:

$P(H_j/E) = K P(H_j) P(E/H_j)$, $j=1,2,\dots,m$;

with $K = 1/P(E)$, $P(E) = \sum_{j=1,2,\dots,m} P(H_j) P(E/H_j)$.

Returning to our example, the H_j would represent the possible hypotheses on the composition of the urn and we, after having given an initial evaluation, we will be able to express a final through the Bayes theorem.

In general, in the objectivist settings of probability (the classical one and the frequentist one), since the probability evaluations are essentially numerical relationships, the observer (the individual who evaluates) has the only task of performing calculations well, based on situations of symmetries or repetitions of a given phenomenon (apparently, however, because in any case precise and subjective choices have been made beforehand: the possible cases considered as equiprobable and the repeated tests considered equiprobable and independent!). In these conceptions the Bayes theorem loses its meaning and its intrinsic value and assumes the role of a purely mathematical result.

In the subjectivist conception of Bruno de Finetti, on the contrary, he best expresses the meaning of how one can (and must) learn from the experience!

It is the same logical process that leads a doctor, to take another example, to make a diagnosis. He, in assessing the presence or absence of a disease in a patient, first visits and expresses a first idea about the disease, then makes him perform a series of diagnostic-instrumental tests, the results of which he uses to establish his presence or not of the disease.

It should be noted, incidentally, that this procedure seems to be part of the logic of the certain, while vice versa more generally it concerns the ambit of the logic of the uncertain: always and only of probability evaluations it is, even if fortunately often the final probabilities of ascertaining the disease are close to 1 or 0 (just think of the fact that in addition to the personal evaluation, clinical trials can also be affected by an error)! In any case, when we go to a doctor, the event: I am suffering from a given disease, it is always possible, neither certain, nor impossible!

Often in applications to estimate the value of some parameter related to a given random phenomenon, the so-called *maximum likelihood method* is used: in short and schematizing it is a matter of evaluating the different values of the probabilities of E (or the probability density, in the case of a

continuous parameter) subject to the hypotheses H_j (the *likelihoods*) and to obtain the maximum value, which is chosen as an *estimation*.

Returning, once again, to our example of the urn of unknown composition we calculate the $P(E/H_j)$, for each j , and then the largest one of all is found; if this, for example, is $P(E/H_3)$ we say that hypothesis H_3 is an estimate of the true composition of the urn. Exemplifying further, in an urn there are 10 balls, of which it is not known how many are red ones. A sampling is carried out, that is a repeated extraction (with restitution) for a total of 5 balls and of these if there are 3 red ones and both E this event. The $P(E/H_j)$, $j = 0, 1, \dots, 10$ are evaluated, obtaining

$$P(E/H) = 5!/3!2! (j^3/10) (10-j)^2/10^2;$$

and distinctly

$$P(E/H_0) = 0$$

$$P(E/H_1) = 0.0081$$

$$P(E/H_2) = 0.0512$$

$$P(E/H_3) = 0.1323$$

$$P(E/H_4) = 0.2304$$

$$P(E/H_5) = 0.3125$$

$$P(E/H_6) = 0.3456$$

$$P(E/H_7) = 0.3087$$

$$P(E/H_8) = 0.2048$$

$$P(E/H_9) = 0.0729$$

$$P(E/H_{10}) = 0 .$$

So, since the maximum value is 0.3456, which is relative to the hypothesis H_6 , it can be deduced that the estimate (of maximum likelihood) for the unknown composition of the urn is 6 red balls out of 10.

This methodology has two contraindications, both logical ones!

The first is to make an estimate relative to hypotheses, using the inverse conditional probabilities $P(E/H_j)$ with respect to those that should be more correctly compared: $P(H_j/E)$; only through these, in a logically unequivocal manner, is it possible to establish which of the hypotheses of composition of the urn is the most probable, observed (supposed to observe) E , and take it as an estimation of the composition!

The second is that in this procedure the probabilities of the hypotheses $P(H_j)$ do not attend, which can always be evaluated and whose non-consideration can be absolutely harmful for the conclusions reached.

In fact, also in the previous example, if we used (as it is necessary) the Bayes theorem, in the evaluation of $P(E/H_j)$ the $P(H_j)$ would enter, and any distribution of these different from the equiprobability, could make maximum the final probability of one of the H_j different from H_6 . If, for some reason, it was known that it is much more likely that as many red balls of another color than the other hypotheses have been placed in the urn, that is in our case we had, for example $P(H_5) = 0.6$, $P(H_4)=P(H_6) = 0.15$, and for simplicity $P(H_0)=P(H_1)=P(H_2)=P(H_3)=P(H_7)=P(H_8)=P(H_9)=P(H_{10}) = 0.0125$, it would be obtained

$$P(H_0/E) = 0$$

$$P(H_1/E) = 0.00035$$

$$P(H_2/E) = 0.00225$$

$$P(H_3/E) = 0.00581$$

$$P(H_4/E) = 0.12185$$

$$P(H_5/E) = 0.66111$$

$$P(H_6/E) = 0.18278$$

$$P(H_7/E) = 0.01357$$

$$P(H_8/E) = 0.00902$$

$$P(H_9/E) = 0.00320$$

$$P(H_{10}/E) = 0 .$$

And so, the hypothesis largely more probable would result H_5 .

I wish to present an even more significant example, for its paradoxical aspects. Paolo does not go to a job interview at the Smile company, we indicate with E this event. The manager of the recruitment department wants to understand why and formulates the following hypotheses:

H_1 = Paul found another job

H_2 = Paul ended up in prison

H_3 = Paul won the lottery or any other reason than the others.

If he were to decide on the basis of the likelihoods, in any case he should have to choose the hypothesis H_2 , since H_2 implies E and therefore $P(E/H_2) = 1$.

While if the Bayes theorem is used, the most probable hypothesis may not be H_2 . In fact, it suffices that they are $P(E/H_1) = 0.6$, $P(E/H_3) = 0.2$ and $P(H_1) = 0.7$, $P(H_2) = 0.25$, $P(H_3) = 0.05$ to obtain

$$P(H_1/E) = 0.618$$

$$P(H_2/E) = 0.368$$

$$P(H_3/E) = 0.014,$$

from which it would be deduced that the most probable hypothesis is the most reasonable!

Bayes theorem, ultimately, is the architrave of the coherence on which the *updating* of probability assessments rests. Through it one can only reduce *uncertainty*, never eliminate it by coming to definite conclusions. From this point of view, it represents an effective form of *non-linear thinking*.

The *updating* of the probability assessments must therefore follow only one principle: that of coherence; this guarantees to *the observer do not violate the rules* in the attribution of probability: between what he had assessed before (the *initial probabilities*) and what he values after (the *final probabilities*). This is when we refer to *learning inductively from experience!*

3. Conclusions

In any case, from all the examples illustrated, it is possible to see how a set of events can be transformed into a *system of events*, when an observer *brings out a coherent* probability assessment for them. As we have seen, in general, of these coherent evaluations there can be more than one, therefore it remains the *responsibility* of the observer to choose the one that he considers to best represent his *state of information*, with respect to the set of events considered!

Consequently, the systemic opening of de Finetti's conception so exerts all its methodological richness and exalts the role of the observer, as the bearer and processor of conscious and precisely responsible choices.

From a systemic point of view, we can say that in this type of inductive logic the interactions between agents (events in this case) must be *coherent*, instead of linear or deductible from each other. In this circumstance, coherence is not something related to the rules of formal logic as in the deduction, for deterministic calculation, but it is relative to the *emergence*.

Coherence, therefore, is not something deterministically calculated and derived: it is designed, learned, experimented and then formalized in the more general construction of models and their simulation, as is also done in the context of *fuzzy logic* (Zadeh et al., 1996).

From this point of view the elements of a system are events.

The observer models the emergent system, which is a configuration of events considered interacting with the probabilities assigned by the observer, as well as by physical interactions.

This approach seems to be necessary for the crucial theoretical role of the observer in emergence processes and their modeling.

Having to do with systems (such as physical, biological and social) considered emerging due to the interactions of the components and assuming objectively that the observer is not part of the system, or is part of it but adopts a logic incompatible with the system considered (that is, assuming a linear logic, thinking of acting in a deterministic space), strategies are used, which although they are not wrong, are at least ineffective.

The approach introduced by de Finetti leads to considering probability systems, starting from single events and arriving at event systems.

In conclusion, it is important to underline how the language and logic of the uncertainty, developed by Bruno de Finetti, play a crucial role in everyday life. Thinking devices that allow us to measure ourselves with *complexity* and produce disciplines that interact systemically, so that concepts, analogies, correspondences and invariants are used consistently.

Tools that until now have allowed us to greatly enrich our knowledge and that if used, as Bruno de Finetti has done not only in the construction of his theory of probabilities but also with regard to his fusionist conception of science and not only, even more they will allow us to increase this enrichment based on overcoming those disciplinary barriers, to which he has always tried to get used to mistrust.

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